Fill in the following identities.

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[a] DIFFERENCE OF ANGLES IDENTITY:

$\cos(x-y) =$ $\cos x \cos y + \sin x \sin y$

[b] SUM OF ANGLES IDENTITY:

$\tan(x+y) =$ $\dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

[c] HALF ANGLE IDENTITY:

$\sin\frac{1}{2}x =$ $\pm\sqrt{\dfrac{1-\cos x}{2}}$

[d] POWER REDUCING IDENTITY:

$\cos^2 x =$ $\frac{1}{2}(1+\cos 2x)$

[e] NEGATIVE ANGLE IDENTITY:

$\sec(-x) =$ $\sec x$

[f] PYTHAGOREAN IDENTITY:

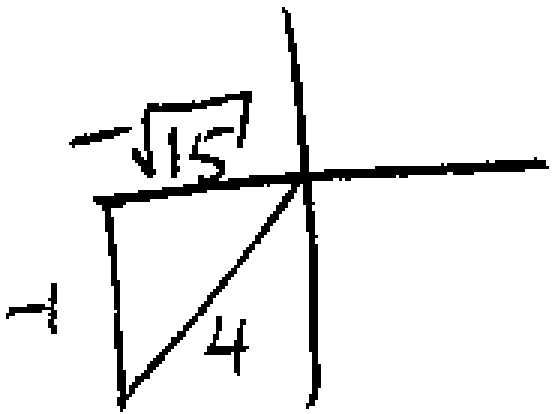
$\cot^2 x =$ $\csc^2 x - 1$

[g] DOUBLE ANGLE IDENTITY:

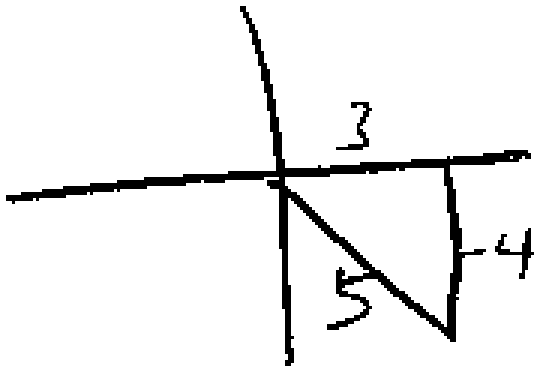
WRITE ALL 3 VERSIONS

$\cos 2x =$ $\cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$

x



y



$\tan 2x$

$$= \frac{2\tan x}{1-\tan^2 x}$$

$$= \frac{2\left(\frac{1}{\sqrt{15}}\right)}{1-\left(\frac{1}{\sqrt{15}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{15}}}{\frac{14}{15}}$$

$$= \frac{2}{\sqrt{15}} \cdot \frac{15}{14}$$

$$= \frac{\sqrt{15}}{7}$$

$$\tan \tfrac{1}{2} x$$

$$= \frac{1 - \cos x}{\sin x}$$

$$= \frac{1 - \left(-\frac{\sqrt{15}}{4}\right)}{-\frac{1}{4}} \cdot \frac{-4}{-4}$$

$$= -4 - \sqrt{15}$$

$$\sin(\underbrace{\arctan(-\tfrac{4}{3})}_{y} - x)$$

$$= \sin y \cos x - \cos y \sin x$$

$$= -\frac{4}{5} \cdot -\frac{\sqrt{15}}{4} - \frac{3}{5} \cdot -\frac{1}{4}$$

$$= \frac{4\sqrt{15}+3}{20}$$

Solve the equation $2(3-\cos\frac{1}{4}x)=5-4\cos\frac{1}{4}x$.

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$6-2\cos\frac{1}{4}x=5-4\cos\frac{1}{4}x$

$2\cos\frac{1}{4}x=-1$

$\cos\frac{1}{4}x=-\frac{1}{2}$

$\frac{1}{4}x=\frac{2\pi}{3}+2n\pi$ or $\frac{4\pi}{3}+2n\pi$, $n\in\mathbb{Z}$

$x=\frac{8\pi}{3}+8n\pi$ or $\frac{16\pi}{3}+8n\pi$, $n\in\mathbb{Z}$

Rewrite $\cos^4 x \sin^2 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE:** _____ / 14 PTS

Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$= \left[\tfrac{1}{2}(1+\cos 2x)\right]^2 \cdot \tfrac{1}{2}(1-\cos 2x)$$

$$= \tfrac{1}{8}(1+\cos 2x)(1+\cos 2x)(1-\cos 2x)$$

$$= \tfrac{1}{8}(1+\cos 2x)(1-\cos^2 2x)$$

$$= \tfrac{1}{8}(1+\cos 2x)(1-\tfrac{1}{2}(1+\cos 4x))$$

$$= \tfrac{1}{8}(1+\cos 2x)(\tfrac{1}{2}-\tfrac{1}{2}\cos 4x)$$

$$= \tfrac{1}{16}(1+\cos 2x)(1-\cos 4x)$$

$$= \tfrac{1}{16} + \tfrac{1}{16}\cos 2x - \tfrac{1}{16}\cos 4x - \tfrac{1}{16}\cos 2x \cos 4x$$

Prove the identity $\cot^2 x + \sec^2 x = \sec^2 x \csc^2 x - 1$.

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$$\hookrightarrow = \sec^2 x (\cot^2 x + 1) - 1$$

$$= \sec^2 x \cot^2 x + \sec^2 x - 1$$

$$= \frac{1}{\cos^2 x} \frac{\cos^2 x}{\sin^2 x} - 1 + \sec^2 x$$

$$= \csc^2 x - 1 + \sec^2 x$$

$$= \cot^2 x + \sec^2 x$$

QED

Solve the equation $4(1+\cos 2x) = 5(1-2\sin x)$ algebraically.

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Round your answers to 4 decimal places.

$4 + 4\cos 2x = 5 - 10\sin x$

$4 + 4(1 - 2\sin^2 x) = 5 - 10\sin x$

$8 - 8\sin^2 x = 5 - 10\sin x$

$0 = 8\sin^2 x - 10\sin x - 3$

$= (4\sin x + 1)(2\sin x - 3)$

$\sin x = -\frac{1}{4}$ OR $\frac{3}{2}$

$x_{REF} = \sin^{-1}\frac{1}{4} \approx 0.2527$

$x \in Q_3$ OR Q_4

$x = \pi + 0.2527 + 2n\pi$

OR $2\pi - 0.2527 + 2n\pi$, $n \in \mathbb{Z}$

$= 3.3943 + 2n\pi$

OR $6.0305 + 2n\pi$, $n \in \mathbb{Z}$